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PIECEWISE PV INVERSION SCHEME IN THE CONTEXT MM5-v3 + Vis5D FOR DIAGNOSTIC AND PROGNOSTIC APPLICATIONS

The numerical modeling of atmospheric circulations is the most powerful tool available to scientists to develop a better physical understanding of the responsible mechanisms and its relation to the weather or the environment. In particular, many studies have used this potential to isolate the role played by different physical factors by means of sensitivity or factor separation techniques. *Boundary factors* (e.g. orography) and *model physics factors* (e.g. condensational latent heat release) have been typically considered.

Comparatively less attention, however, has been paid to the effects of internal features of the flow dynamics (jet streaks, troughs, fronts, etc.) probably because, unlike the boundary or model physics factors, modifying or switching off these elements in the simulations is not straightforward. The three-dimensional nature and mutual dependence of pressure, temperature and wind fields pose serious constraints on the ways these fields can be altered without compromising the delicate dynamical balances that govern both the model equations and actual data. A relatively clean approach to deal with the *dynamical factors* is based on the concept of potential vorticity (PV) and its invertibility principle (Hoskins et al. 1985)[1]. According to this principle, given some balanced flow constraints and proper boundary conditions for the meteorological fields (pressure, temperature and wind), the knowledge of the three-dimensional distribution of PV can be used to infer the balanced meteorological fields. Application of *piecewise PV inversion* is particularly useful, since once identified, any PV element of interest as well as its associated mass and wind fields can be isolated in a consistent way for diagnostic and prognostic purposes (e.g. Romero 2001)[2].

The piecewise PV inversion technique of Davis and Emanuel (1991)[3], based on the Charney (1955)[4] nonlinear mass-wind balance equation, has been implemented as a set of Vis5D external analysis functions that can be applied to MM5-v3 output data. The balanced flow associated with an instantaneous distribution of Ertel's PV and a balanced reference state associated with some background Ertel's PV distribution are solved iteratively; then, the contribution to the balanced flow due to any user-defined PV anomaly ***i*** is found by means of another iterative process (see the system of equations in next page). Since the system is solved step by step using the Vis5D external functions capability, the piecewise PV inversion process is fully interactive and the results can be continuously visualized.

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[1] Hoskins, B. J, McIntyre, M. E. and Robertson, A. W., 1985: On the use and significance of isentropic potential vorticity maps. *Quart. J. Roy. Meteor. Soc.*, **111**, 877-946.

[2] Romero, R., 2001: Sensitivity of a heavy rain producing Western Mediterranean cyclone to embedded potential vorticity anomalies. *Quart. J. Roy. Meteor. Soc.*, **127**, 2559-2597.

[3] Davis, C. A. and Emanuel, K. A., 1991: Potential vorticity diagnostics of cyclogenesis. *Mon. Wea. Rev.*, **119**, 1929-1953.

[4] Charney, J. G., 1955: The use of primitive equations of motion in numerical prediction. *Tellus*, **7**, 22-26.

PIECEWISE PV INVERSION TECHNIQUE
(Davis and Emanuel; *MWR* 1991)

1) Balanced flow (ϕ , ψ) given instantaneous distribution of Ertel's PV (q):

*** Charney (1955) nonlinear balance equation**

$$\nabla^2\phi = \boldsymbol{\nabla}\cdot f\boldsymbol{\nabla}\psi + 2m^2\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2} - \left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right]$$

f Coriolis parameter m map-scale factor

*** Approximate form of Ertel's PV**

$$q = \frac{g\kappa\pi}{p}\left[(f + m^2\nabla^2\psi)\frac{\partial^2\phi}{\partial\pi^2} - m^2\left(\frac{\partial^2\psi}{\partial x\partial\pi}\frac{\partial^2\phi}{\partial x\partial\pi} + \frac{\partial^2\psi}{\partial y\partial\pi}\frac{\partial^2\phi}{\partial y\partial\pi}\right)\right]$$

p pressure g gravity $\kappa = Rd/Cp$ $\pi = Cp(p/po)^\kappa$

*** Boundary conditions** Lateral (Dirichlet) / Top and Bottom(Neumann): $\partial\phi/\partial\pi = f\partial\psi/\partial\pi = -\theta$

θ potential temperature

2) Reference state: Balanced flow ($\bar{\phi}$, $\bar{\psi}$) given time mean distribution of Ertel's PV ($\bar{q}$):

*** Same equations as in 1), except using time mean fields instead of instantaneous fields**

3) Perturbation fields (ϕ' , ψ' , q') given by the definitions: $(q,\phi,\psi) = (\bar{q},\bar{\phi},\bar{\psi}) + (q',\phi',\psi')$

PIECEWISE PV INVERSION TECHNIQUE
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4) We consider that q' is partitioned into N portions or anomalies: $q' = \sum_{n=1}^{N} q_n$

5) Piecewise inversion: (ϕ_n , ψ_n) associated with q_n?   **… and requiring:** $\phi' = \sum_{n=1}^{N}\phi_n$ $\psi' = \sum_{n=1}^{N}\psi_n$

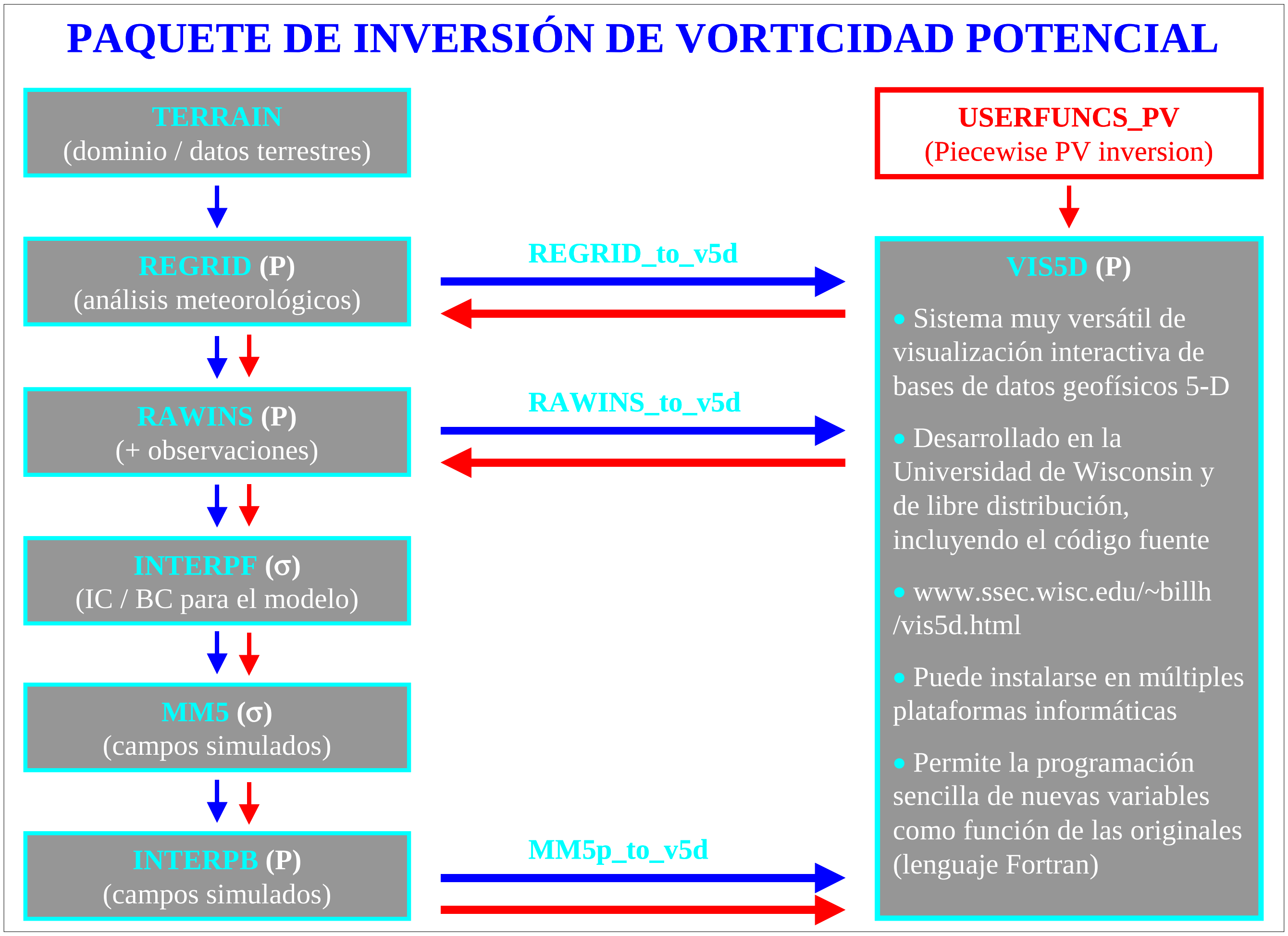
…After substitution of the above summations in the balance and PV equations and some rearrangements of the nonlinear terms:

$$\nabla^2\phi_n = \boldsymbol{\nabla}\cdot f\boldsymbol{\nabla}\psi_n + 2m^2\left(\frac{\partial^2\psi^*}{\partial x^2}\frac{\partial^2\psi_n}{\partial y^2} + \frac{\partial^2\psi^*}{\partial y^2}\frac{\partial^2\psi_n}{\partial x^2} - 2\frac{\partial^2\psi^*}{\partial x\partial y}\frac{\partial^2\psi_n}{\partial y\partial x}\right)$$

$$\begin{aligned} q_n \;=\; & \frac{g\kappa\pi}{p}\Bigg[(f + m^2\nabla^2\psi^*)\frac{\partial^2\phi_n}{\partial\pi^2} + m^2\frac{\partial^2\phi^*}{\partial\pi^2}\nabla^2\psi_n \\ & -m^2\left(\frac{\partial^2\phi^*}{\partial x\partial\pi}\frac{\partial^2\psi_n}{\partial x\partial\pi} + \frac{\partial^2\phi^*}{\partial y\partial\pi}\frac{\partial^2\psi_n}{\partial y\partial\pi}\right) - m^2\left(\frac{\partial^2\psi^*}{\partial x\partial\pi}\frac{\partial^2\phi_n}{\partial x\partial\pi} + \frac{\partial^2\psi^*}{\partial y\partial\pi}\frac{\partial^2\phi_n}{\partial y\partial\pi}\right)\Bigg] \end{aligned}$$

where $()^* = \overline{()} + \frac{1}{2}()'$ **Boundary conditions:** Lateral (homogeneous) / Top and bottom (using θ_n)

According to the procedure shown below, in the *diagnostic mode* the piecewise PV inversion scheme can be applied on the isobaric meteorological fields generated by REGRID or RAWINS (pre-simulation fields) or by INTERPB (simulated fields). In the *prognostic mode*, the piecewise PV inversion scheme is applied on the pre-simulation fields generated by REGRID or RAWINS, and the resulting balanced fields are used to modify the pre-simulation fields in order to design a simulation with perturbed initial conditions (an example from Romero 2001 is shown in next page).



A number of Vis5D external analysis functions (FORTRAN programs) must be called in sequence to apply the piecewise PV inversion. In the table, a typical sequence necessary to design a simulation with perturbed initial conditions is described (the **bold** functions must always be called, the other functions are not required but they help to follow the PV inversion process). To run the piecewise PV inversion scheme, remember to have in the Vis5D variables list the following fields:

`T(°C)`, `U(m s`$^{-1}$`)`, `V(m s`$^{-1}$`)` and `H(m)` (instantaneous fields defining the atmospheric state)
`Tm(°C)`, `Um(m s`$^{-1}$`)`, `Vm(m s`$^{-1}$`)` and `Hm(m)` (time-mean fields, or another reference state)
`CORIOLIS(s`$^{-1}$`)`, `MAPFACCR(no dimensions)`, `TERRAIN(m)` (other necessary fields)

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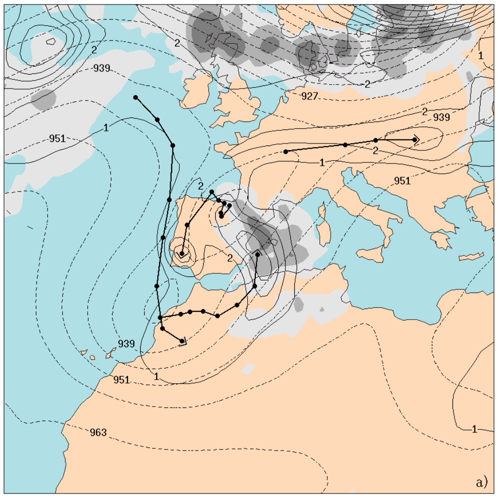
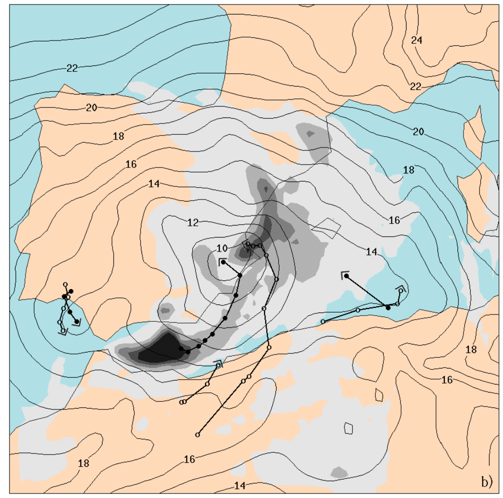
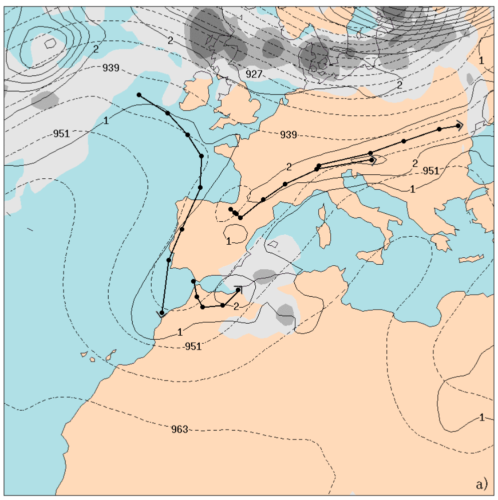
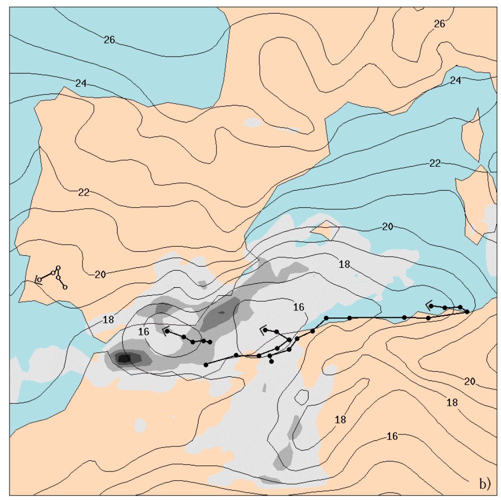


CONTROL SIMULATION





UPPER-LEVEL PV ANOMALIES REMOVED



Function	Physical meaning	Units	Actions needed
Tp	Temperature PERTURBATION (= T – Tm)	°C	-
Up Vp	Wind PERTURBATION (= U – Um / = V – Vm)	m s^{-1}	-
Hp	Geopotential height PERTURBATION (= H – Hm)	m	-
ErPV	Ertel potential vorticity (INSTANTANEOUS state)	PV units	-
ErPVm	Ertel potential vorticity (REFERENCE state)	PV units	-
ErPVp	Ertel potential vorticity (PERTURBATION = ErPV - ErPVm)	PV units	-

ErPVi i=1,2,3	Ertel potential vorticity (ANOMALY **i** of perturbation ErPVp)	PV units	Define in subroutine
H0	First guess for I_H	m	-
I_H	Inverted (balanced) geopotential height (INSTANTANEOUS state)	m	-
S0	First guess for I_S	$m^2\ s^{-1}$	-
I_S	Inverted (balanced) streamfunction (INSTANTANEOUS state)	$m^2\ s^{-1}$	-
I_T	Inverted (balanced) temperature (INSTANTANEOUS state)	°C	-
I_U I_V	Inverted (balanced) wind (INSTANTANEOUS state)	$m\ s^{-1}$	-
Hm0	First guess for I_Hm	m	-
I_Hm	Inverted (balanced) geopotential height (REFERENCE state)	m	-
Sm0	First guess for I_Sm	$m^2\ s^{-1}$	-
I_Sm	Inverted (balanced) streamfunction (REFERENCE state)	$m^2\ s^{-1}$	-
I_Tm	Inverted (balanced) temperature (REFERENCE state)	°C	-
I_Um I_Vm	Inverted (balanced) wind (REFERENCE state)	$m\ s^{-1}$	-
Hp0	= H0 – Hm0	m	-
I_Hp	Inverted (balanced) geopotential height (PERTURBATION = I_H – I_Hm)	m	-
Sp0	= S0 – Sm0	$m^2\ s^{-1}$	-
I_Sp	Inverted (balanced) streamfunction (PERTURBATION = I_S – I_Sm)	$m^2\ s^{-1}$	-
I_Tp	Inverted (balanced) temperature (PERTURBATION = I_T – I_Tm)	°C	-
I_Up I_Vp	Inverted (balanced) wind (PERTURBATION = I_U – I_Um / = I_V – I_Vm)	$m\ s^{-1}$	-
TBpi i=1,2,3	Bottom boundary condition for I_Tp**i** (as ANOMALY **i** of perturbation I_Tp)	°C	Define in subroutine
TTpi i=1,2,3	Top boundary condition for I_Tp**i** (as ANOMALY **i** of perturbation I_Tp)	°C	Define in subroutine

I_Hpi i=1,2,3	Inverted (balanced) geopotential height (associated with ANOMALY ErPV**i**)	m	Define B.C. in subroutine
I_Spi i=1,2,3	Inverted (balanced) streamfunction (associated with ANOMALY ErPV**i**)	$m^2\ s^{-1}$	Define B.C. in subroutine
I_Tpi i=1,2,3	Inverted (balanced) temperature (associated with ANOMALY ErPV**i**)	^{o}C	-
I_Upi I_Vpi i=1,2,3	Inverted (balanced) wind (associated with ANOMALY ErPV**i**)	$m\ s^{-1}$	-
I_REG	Substitutes REGRID_DOMAIN meteorological fields by I_Hp, I_Tp, IUp and I_Vp	-	-
I_REGpi i=1,2,3	Modifies REGRID_DOMAIN meteorological fields using I_Hp**i**, I_Tp**i**, IUp**i** and I_Vp**i**	-	Define input and output REGRID_DOMAIN files in subroutine, and type of modification
I_RAW	Substitutes RAWINS_DOMAIN meteorological fields by I_Hp, I_Tp, IUp and I_Vp	-	-
I_RAWpi i=1,2,3	Modifies RAWINS_DOMAIN meteorological fields using I_Hp**i**, I_Tp**i**, IUp**i** and I_Vp**i**	-	Define input and output RAWINS_DOMAIN files in subroutine, and type of modification